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Required Repair Time to Assure the Given/Specified Availability

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Abstract

A novel, simple, easy-to-use, flexible and physically meaningful methodology is suggested for the assessment of the required repair/restoration time, so that the object's/system's availability is swiftly restored, thereby keeping this availability on the specified/desirable/required level during the entire time of the system's operation. A working table for the time-dependent availability function is obtained for the following two major governing input variables: 1) the product of the anticipated failure rate of the system of interest and the time of operation and 2) the ratio of the intensity of the restoration process to the mean-time-to failure (MTTF). This intensity is simply reciprocal to the mean time to repair (MTTR). The general concept is illustrated by a practical example. Several extensions of this work are considered and indicated, and particularly the role of the human-system interaction ("human-in-the-loop") situations, when system's reliability and human performance contribute jointly to the never-100%-failure-free operation process.

Introduction

One important characteristic of the operational reliability of a complex object or a system, such as, e.g., air- or space-craft, moto- or a maritime vehicle, long-haul communication system, military equipment, an electronic or a photonic product, medical instrumentation, all kinds of robots, etc., is its availability [1-9]. Availability is defined, when a probabilistic approach is used, as the probability that the object/system of importance is in the sound condition and is therefore available to the user, when needed. This probability could be kept sufficiently high by assuring high level of the object's reparability, which is characterized, first of all, after failure occurs, by the restoration time. The availability level depends on its reparability ("serviceability"), which is the ease, with which the system can be maintained and repaired.

The objective of the analysis that follows is to develop a simple, easy-to-use and physically meaningful methodology that enables to quantify, on the probabilistic basis, the role of the random time-to-repair (TTR) vs. (also random) time-to-failure (TTF). The methodology is not restricted to a particular repairable object or a system and, in the authors' opinion, is applicable to a wide variety of engineering products and applications. It is assumed, however, in this analysis that the failure event always follows the Poisson's distribution and the restoration time always follows an exponential distribution. Possible deviations from these distributions and the effect of such deviations on the obtained data are considered as future work. It is noteworthy also that in practical applications of the suggested methodology it might not be easy to determine, as the major input parameters, the failure rate and the restoration time. This is because these parameters are often time-varying. Possible approaches of handling this circumstance are considered as future work as well.

Analysis

Failures are rare events, and therefore the process of failures and restorations could be characterized by a constant failure rate λ (steady-state portion of the bathtub curve), and the probability of occurrence of n failures during the time t follows the Poisson's distribution (see, e.g., [4])

$$P_n(t) = \frac{(\lambda t)^n}{n!} e^{-\lambda t} . \quad (1)$$

When restorations are carried out swiftly and the number of the necessary repairs reduces with an increase in their duration, one can assume that the restoration time t is an exponentially distributed random variable, so that its probability density distribution function is

$$f(t) = \mu e^{-\mu t}, \quad (2)$$

where the intensity $\mu = \frac{1}{\prec t \succ}$ of the restoration process is reciprocal to its mean value $\prec t \succ$. But how "swiftly" is swiftly enough to keep the system's availability sufficiently high?

Let $K(t)$ be the probability that the product is in the working condition and $k(t)$ is the probability that it is idle. When considering random processes with discrete states and continuous time, it is assumed that the transitions of the system S from the state s_i to the state s_j are defined by transitional probabilities λ_{ij} . If the governing flow of events is of Poisson's type, the random process is a Markovian process. The probability of state $p_i(t) = P\{S(t) = s_i\}, i = 1,2,...,n$ of such a process, i.e., the probability that the system S is in the state s_i at the moment of time t , can be found from the Kolmogorov's equation (see, e.g., [4])

$$\frac{dp_i(t)}{dt} = \sum_{j=1}^{n} \lambda_{jl} p_j(t) - p_i(t) \sum_{j=1}^{n} \lambda_{ij},$$

$i = 1,2,...,n$ (3)

Applying this equation to the processes (1) and (2), the following equations for the probabilities $K(t)$ and $k(t)$ can be obtained:

$$\frac{dK(t)}{dt} = \mu k(t) - \lambda K(t),$$
$$\frac{dk(t)}{dt} = \lambda K(t) - \mu k(t). \quad (4)$$

The probability normalization condition requires that the relationship $K(t) + k(t) = 1$ takes place for any moment t of time. Then the probabilities $K(t)$ and $k(t)$ in the equations (4) can be separated:

$$\frac{1}{\lambda+\mu}\frac{dK(t)}{dt} + K(t) = \frac{\mu}{\lambda+\mu},$$
$$\frac{1}{\lambda+\mu}\frac{dk(t)}{dt} + k(t) = \frac{\lambda}{\lambda+\mu}. \quad (5)$$

These equations have the following solutions:

$$K(t) = C\exp[-(\lambda+\mu)t] + \frac{\mu}{\lambda+\mu},$$
$$k(t) = C\exp[-(\lambda+\mu)t] + \frac{\lambda}{\lambda+\mu}. \quad (6)$$

The constant C of integration is determined from the initial conditions depending on whether the item is in the working or in an idle condition at the initial moment of time. If it is in the working condition, the initial conditions $K(0) = 1$ and $k(0) = 0$ should be used, and $C = \frac{\lambda}{\lambda+\mu}$. If the item is in the idle condition, the initial conditions $K(0) = 0$ and $k(0) = 1$ should be applied, and $C = \frac{\mu}{\lambda+\mu}$. Hence, the availability function is

$$K(t) = \frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu}\exp[-(\lambda+\mu)t], \quad (7)$$

if the item is in the working condition at the initial moment of time, and is $K(t) = \frac{\mu}{\lambda+\mu} - \frac{\mu}{\lambda+\mu}\exp[-(\lambda+\mu)t]$, (8)

if the item is in the idle condition at the initial moment of time. The constant (steady-state) part

$$K = \frac{\mu}{\lambda+\mu} = \frac{1}{1+\frac{\lambda}{\mu}} = \frac{1}{1+\frac{\prec t_r \succ}{\prec t_f \succ}} = \frac{1}{1+\frac{MTTR}{MTTF}} \quad (9)$$

of the equations (7) and (8) is known as availability index. It determines the percentage of time, in which the item is in workable (available) condition. In the formula (9), $MTTF = \prec t_f \succ = \frac{1}{\lambda}$ is the mean time to failure, and $MTTR = \prec t_r \succ = \frac{1}{\mu}$ is the mean time to repair. If the system consists of many items, the formula (9) can be generalized as follows:

$$K = \frac{1}{1+\sum_{l=1}^{n}\frac{\prec t_r \succ_i}{\prec t_f \succ_i}} \quad (10)$$

The relationship (7) is tabulated in Table 1 and plotted in Fig.1. When the failure rate λ is low (the MTTF is high) and the restoration rate μ is high (the MTTR is low), then the ratios $\frac{\mu}{\lambda}$ are significant. This leads to a high steady-state availability index and to a short transition time to it. When the ratio $\frac{\mu}{\lambda}$ is small (long MTTR and short MTTF), the transition period could be significant. For zero $\frac{\mu}{\lambda}$ ratios (non-repairable system: MTTR is infinitely long), the formula (7) yields:

$$K(t) = \exp[-(\lambda t)], \tag{11}$$

i.e., the availability index is not different from the probability of non-failure (dependability) of a non-repairable item. The Table 1 and Fig.1 quantify the role of the $\frac{\mu}{\lambda}$ ratio.**Table 1.** Calculated availability function $K(t)$

λt	**0**	**0.1**	**0.2**	**0.5**	**1.0**	**2.0**	**3.0**	**4.0**	**5.0**	∞
μ / λ	x	x	x	x	x	x	x	x	x	x
0	1.0	0.9048	0.8187	0.6065	0.3679	0.1353	0.0498	0.0153	0.00674	0
0.2	1.0	0.9058	0.8222	0.6240	0.4177	0.2423	0.1894	0.1735	0.1687	0.1666
0.4	1.0	0.9067	0.8256	0.6404	0.4618	0.3291	0.2964	0.2860	0.2864	0.2857
0.6	1.0	0.9076	0.8288	0.6558	0.5012	0.4005	0.3801	0.3760	0.3752	0.3750
0.8	1.0	0.9085	0.8320	0.6702	0.5362	0.4596	0.4469	0.4448	0.4445	0.4444
1.0	1.0	0.9094	0.8352	0.6839	0.5677	0.5092	0.5012	0.5002	0.5000	0.5000
2.0	1.0	0.9136	0.8496	0.7410	0.7118	0.6675	0.6667	0.6666	0.6666	0.6666
3.0	1.0	0.9176	0.8623	0.7838	0.7546	0.7501	0.7500	0.7500	0.7500	0.7500
4.0	1.0	0.9213	0.8020	0.8164	0.8013	0.8000	0.8000	0.8000	0.8000	0.8000
5.0	1.0	0.9248	0.8835	0.8416	0.8337	0.8333	0.8333	0.8333	0.8333	0.8333
6.0	1.0	0.9281	0.8924	0.8615	0.8572	0.8571	0.8571	0.8571	0.8571	0.8571
10.0	1.0	0.9393	0.9192	0.9095	0.9090	0.9090	0.9090	0.9090	0.9090	0.9090
20.0	1.0	0.9582	0.9531	0.9524	0.9524	0.9524	0.9524	0.9524	0.9524	0.9524
50.0	1.0	0.9805	0.9804	0.9804	0.9804	0.9804	0.9804	0.9804	0.9804	0.9804
100.0	1.0	0.9901	0.9901	0.9901	0.9901	0.9901	0.9901	0.9901	0.9901	0.9901
1000.0	1.0	0.9990	0.9990	0.9990	0.9990	0.9990	0.9990	0.9990	0.9990	0.9990
10000.0	1.0	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
∞	1.0000									

If one intends to determine the ratio $\bar{\mu} = \frac{\mu}{\lambda}$ for the given product λt of the failure rate and time) and the availability function $K(t)$ from the transcendental equation (7), then the following recurrent equation, based on the Newton's formula, can be used:

$$\bar{\mu}_{n+1} = \bar{\mu}_n - (1+\bar{\mu}_n)\frac{[\bar{\mu}_n + \exp(-(1+\bar{\mu}_n)\lambda t)] - K(t)(1+\bar{\mu}_n)}{1-[1+(1+\bar{\mu}_n)\lambda t]\exp(-(1+\bar{\mu}_n)\lambda t)}, \quad n = 0,1,2,... \tag{12}$$

For a steady-state situation, with large λt values, this formula can be simplified:

$$\bar{\mu}_{n+1} = (1+\bar{\mu}_n)^2 K(t) - \bar{\mu}_n^2 \tag{13}$$

Let the restoration time τ be a random variable that is distributed in accordance of the Raleigh law:

$$f_\tau(\tau) = \frac{\tau}{\tau_0^2}\exp\left(-\frac{\tau^2}{2\tau_0^2}\right). \tag{14}$$

Here τ_0 is the most likely restoration time, which is the maximum value of the probability density distribution function $f_\tau(\tau)$. The rationale behind the suitability of such a distribution is that the most likely restoration time should be short; the probability of zero restoration times should be zero, and so should be very long restoration times; the variable of interest (time) is always positive, and that the distribution should be heavily skewed in the direction of short times. The formula (14) indicates that the probability that the random restoration time exceeds a certain level $\hat{\tau}$ can be evaluated as

$$P_\tau(\hat{\tau}) = P(\tau \succ \hat{\tau}) = \exp\left(-\frac{\hat{\tau}^2}{2\tau_0^2}\right) \quad . \tag{15}$$

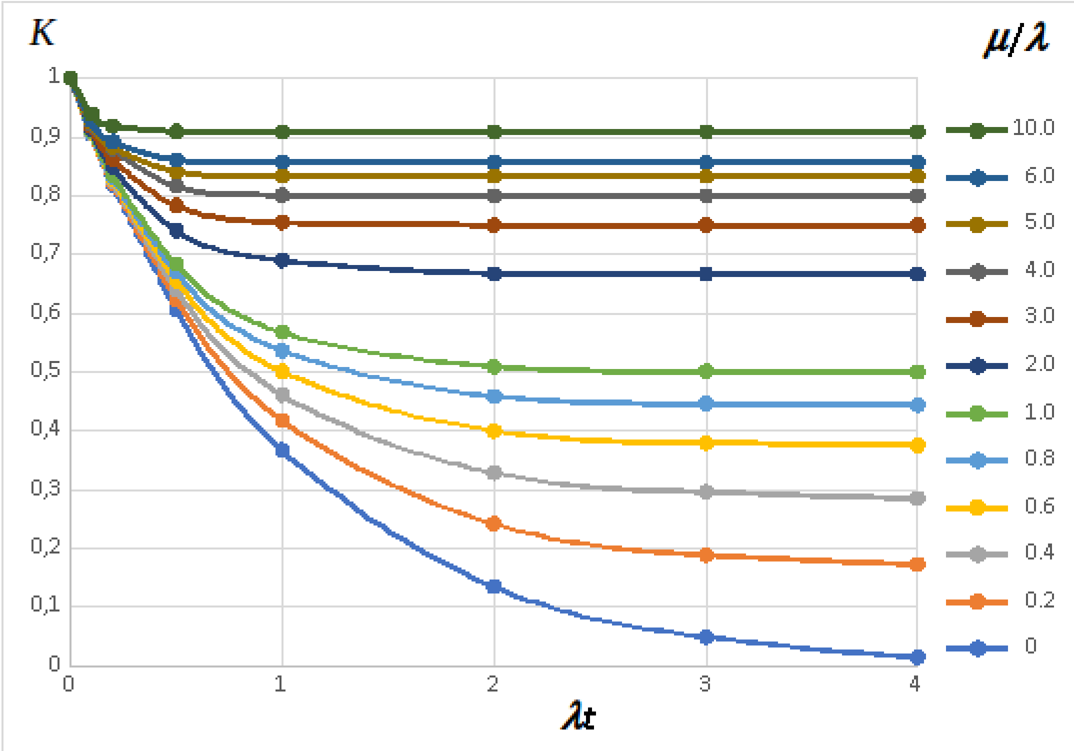

Fig.1. Availability function $K(t)$ vs the product λt of failure rate and time for different restoration-to-failure-rate ratios μ / λ. The availability function is higher and stabilizes faster for high μ / λ ratios, i.e., for large MTTF and low MTTR.

Clearly, this probability should be low for low restoration times and high availability.
Using the equation (7), we obtain the following relationship between the level K_* of the availability function $K(t)$ and the probability $P(K_*)$ that this level is exceeded (reliability criterion):

$$K_* = \frac{1+\tau_* \exp\left(-\frac{1+\tau_*}{\tau_*}\lambda t\right)}{1+\tau_*}. \tag{16}$$

Here

$$\tau_* = \lambda\tau_0\sqrt{-2\ln[P(K_*)]} \tag{17}$$

is the level of the corresponding dimensionless restoration time. The threshold τ_* changes from zero to infinity, and the availability threshold K_* and the probability $P(K_*)$ change from one to zero. Thus, high probabilities of non-failure, as far as high enough availability is concerned, are characterized by low thresholds τ_*. The threshold τ_* should be kept low for high availability. The formula (17) indicates that lower products of the failure rate and most likely TTR, and high $P(K_*)$ values keep the threshold τ_* of the TTF low.

If the product λt is significant (steady-state situation, significant time t), the formula (16) can be simplified:

$$K_* = \frac{1}{1+\tau_*}. \tag{18}$$

By putting $\frac{\mu}{\lambda} = \frac{1}{\tau_*}$ in Table 1 and in Fig.1 one can use this table and figure to assess the K_* level for the given failure rate λ, the most likely restoration time τ_0, the probability $P(K_*)$ and the time t in operation.

Numerical Example

Let, e.g., the steady-state failure rate of the system defined by the bathtub curve is $\lambda = 10^{-3}$ 1/hrs, and the reduced, but still acceptable, level of the availability function in steady-state operations, because of the detected malfunction in the system, should not be below $K(t) = 0.95$. Table 1 and Fig.1 indicate that the corresponding steady-state $\bar{\mu} = \frac{\mu}{\lambda}$ value should be as high as about 20. This means that the corresponding MTTR should not exceed $MTTR = \frac{1}{\mu} = \frac{1}{\bar{\mu}\lambda} = \frac{1}{20x10^{-3}} = 50$ hours. A more accurate number for the $\bar{\mu} = \frac{\mu}{\lambda}$ ratio of the restoration-to-failure rates could be obtained using the formula (13) and $\bar{\mu} = 20$ value as zero approximation:

$$\bar{\mu}_1 = (1+\bar{\mu}_0)^2 K(t) - \bar{\mu}_0^2 = 21^2 x0.95 - 20^2 = 18.95,$$

$$\bar{\mu}_2 = (1+\bar{\mu}_1)^2 K(t) - \bar{\mu}_1^2 = 19.95^2 x0.95 - 18.95^2 = 18.95 = 19.00,$$

$$\bar{\mu}_3 = (1+\bar{\mu}_2)^2 K(t) - \bar{\mu}_2^2 = 20.0^2 x0.95 - 19.00^2 = 19.00.$$

Then the more accurate prediction, as far as the required MTTR is concerned, is

$$MTTR = \frac{1}{\mu} = \frac{1}{\bar{\mu}\lambda} = \frac{1}{19x10^{-3}} = 52.6 \text{ hours.}$$

Using this number as the most likely MTTR τ_0 and the formula (18) with $K_* = 0.95$, we conclude that the non-dimensional threshold value (the lower the better), of the most likely MTTR should be

$$\tau_* = \frac{1}{K_*} - 1 = \frac{1}{0.95} - 1 = 0.052632$$

for high enough availability. Then the probability $P(K_*)$ can be found as

$$P(K_*) = \exp\left[-\frac{1}{2}\left(\frac{\tau_*}{\lambda\tau_0}\right)^2\right] = \exp\left[-\frac{1}{2}\left(\frac{0.052632}{10^{-3}x19}\right)^2\right] = 0.02156,$$

and is not high at all. This should be attributed to the application of the Rayleigh based distribution. The situation might be different with other distributions, like, e.g., Weibull distribution for the TTR.

Conclusions

The following major conclusions are drawn from the carried out analysis:

- A simple and easy-to-use methodology is suggested to quantify, on the probabilistic basis, the role of the random time-to-repair (TTR) in connection with the (also random) time-to-failure (TTF) to keep the availability of the product on the acceptable (and high enough) level.
- Several extensions of this work are considered, and particularly the role of the human-system interaction.
- The methodology is not restricted to a particular repairable object or a system and, in the authors' opinion, is applicable to a wide variety of engineering products and applications.
- Since it is assumed in this analysis that the failure event always follows the Poisson's distribution and the restoration time always follows an exponential distribution, possible deviations from these distributions and the effect of such deviations on the obtained data is considered as future work.
- Because in practical applications of the suggested methodology it might not be easy to determine, the failure rate and the restoration time as the major input parameters, possible approaches of handling this circumstance are considered as future work as well.

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